Easily tuneable nonlinear optical loop mirror including low-birefringence, highly twisted fibre with invariant output polarisation

O. Pottiez a,b,*, E.A. Kuzin a, B. Ibarra-Escamilla a, F. Méndez Martínez a

a Instituto Nacional de Astrofísica, Óptica y Electrónica (INAOE), L.E. Erro 1, Tonantzintla 72000, Puebla, Pue., Mexico
b Scientific Research Worker of FNRS, Belgian Fund for Scientific Research, Service d’Electromagnétisme et de Télécommunications, Faculté Polytechnique de Mons, Boulevard Dolez 31, B-7000 Mons, Belgium

Received 17 June 2003; received in revised form 3 October 2003; accepted 17 October 2003

Abstract

We investigate theoretically the operation of a versatile nonlinear optical loop mirror (NOLM) structure to be used in optical communication systems. The proposed device is a fibre Sagnac interferometer that includes a low-birefringence, highly twisted fibre and a quarter-wave plate retarder in the loop. We study, both analytically and numerically, the evolution of the intensity-dependent NOLM transmission for both output polarisation components, using different models for the NOLM. From this analysis, we propose an easy way to adjust the position of the NOLM maximum transmission, simply by tuning the angle of the retarder. This procedure is particularly useful for amplitude regularisation of an optical signal. We also demonstrate that, if a tuneable optical attenuator is inserted in the loop, the positions of both maximum and minimum transmission can be tuned separately, using a perfectly reproducible procedure. It is therefore possible to optimise the NOLM transmission for both pedestal and amplitude fluctuations removal in an optical pulse train. For a circular input polarisation, this procedure ensures the highest possible contrast between minimum and maximum transmission, and an output polarisation state that is linear and independent of the input power. Finally, we demonstrate that the transmission characteristic of this NOLM is robust to environmentally induced changes in the fibre birefringence. Thanks to its versatility, robustness and polarisation invariance, this device is thought to be of primary interest for applications such as passive mode locking, pulse compression and pedestal suppression, amplitude regularisation in harmonically mode-locked, rational-harmonically mode-locked or subharmonic synchronous mode-locked lasers, as well as damping of relaxation oscillations.

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PACS: 07.60.Vg; 42.65.Pc; 42.81.Gs

Keywords: Sagnac interferometer; Fibre-optic devices; Optical communications

* Corresponding author. Tel.: +52-222-266-3100-1213; fax: +52-222-247-2940.
E-mail address: pottiez@inaoep.mx (O. Pottiez).
1. Introduction

Fibre Sagnac interferometers, also called non-linear optical loop mirrors (NOLMs) [1], have long been proposed for applications like optical switching [2], demultiplexing [3], or passive mode locking [4]. These applications rely on the saturable absorber characteristic of the interferometer, which shows up very low transmission at low input power, whereas it becomes completely transparent for some higher value of the input power. Depending on the applications, the mirror may be saturated by the signal itself, or by a control beam. More recently, this class of devices has been considered for regularising the characteristics of optical signals, in particular their amplitude. As an example, the reduction of a strong amplitude modulation affecting optical pulse trains was demonstrated using a NOLM in a single-pass configuration [5], or inserted in a resonant laser structure [6]. In a continuous-wave fibre laser, such a fibre interferometer was used to damp relaxation oscillations, leading to their complete suppression [7]. Moreover, when a high-quality optical clock is available, a NOLM was shown able to remove not only amplitude fluctuations, but also timing jitter, chirp and pulse profile irregularities from a highly deteriorated input data stream [8]. Finally, it has to be mentioned that NOLMs were also successfully demonstrated for pedestal suppression in optical pulse trains, an effect that complements the well-known pulse compression effect [9–11].

For these applications, in particular for amplitude regularisation and pedestal suppression, an accurate control of the positions of minimum and maximum NOLM transmission is recommended. As an example, Fig. 1 illustrates the effect of a sinusoidal transmission characteristic on a train of optical pulses affected by amplitude noise and accompanied by a substantial level of background radiation. Whereas maximum amplitude noise reduction and pedestal suppression are obtained for some position of the transmission curve (Fig. 1(a)), incomplete pedestal elimination and amplification of the amplitude noise may occur when this curve departs from the optimal situation (Fig. 1(b)). In practice, an optimal configuration of the NOLM (leading to an optimal transmission characteristic) is usually obtained by tuning a polarisation controller (PC) that is inserted in the low-birefringence fibre loop. This PC locally modifies the fibre birefringence and, thereby, the polarisation of the beams propagating in the loop, which in turn affects the intensity-dependent characteristic of the NOLM [12]. Unfortunately, neither the exact birefringence profile of the fibre (which is a function of its local structure and stress), nor its total birefringence including the effect of the PC in a given position is usually known with precision, so that this
adjustment procedure is purely empirical and hardly reproducible. As a matter of fact, most authors who are dealing with fibre Sagnac interferometers for various purposes are able to obtain a transmission characteristic adapted to their particular application, but they usually provide very poor information on the exact manipulations required to reach this result. Moreover, to the fixed birefringence of the fibre must be added a significant time-dependent contribution, which changes with environmental conditions, like in particular temperature. Such modifications of the NOLM birefringence are likely to generate a slow drift of the transmission characteristic, which constrains to a periodic readjustment of the PC for optimal NOLM operation to be maintained. Finally, it is important to stress that, in a conventional NOLM, the output polarisation is generally not kept constant along the nonlinear transmission characteristic, even for a fixed input polarisation. In other words, a change in the input power modifies not only the NOLM power transmission, but also the polarisation state of the output beam (which changes with environmental conditions as well). This aspect, which has received very few consideration so far in the literature, is important however when the NOLM is used in combination with single-polarisation, polarisation-sensitive or polarisation-maintaining optical devices.

In this paper, we investigate theoretically a particularly promising NOLM structure, namely a fibre Sagnac interferometer that includes a low-birefringence, highly twisted fibre in the loop and a quarter-wave plate retarder (QWP) after one of the coupler output ports (Fig. 2, with \( L_1 = L_2 = 1 \)). Using an analysis proposed in [13], we bring out its main properties, like its easy tuneability, robustness to environmental perturbations and, in certain conditions, intensity-invariant output polarisation. All the qualities of this NOLM make it a very promising device in particular for applications such as pulse shortening and pedestal suppression, amplitude regularisation and relaxation oscillations damping.

2. NOLM transmission in the linear regime

The intensity-dependent transmission characteristic of a NOLM is founded on the nonlinear phase shift between the counter-propagating light beams in the loop, generally attributed to self-phase modulation (SPM). However, such a phase shift may appear only if the NOLM symmetry is broken in some way. Considering SPM only, this is most frequently realised through the use of an asymmetric coupler in the NOLM design: the difference in power between the counter-propagating signals is then responsible for the nonlinear phase shift. In contrast, the NOLM structure described above, which was first proposed by Kuzin et al. [13], behaves in a noticeably different manner. In this case, the asymmetric element is a QWP inserted in the loop, after one of the coupler output ports. This QWP makes cross-phase modulation (XPM) play an important role in the set-up of the NOLM transmission. As a consequence, even when SPM does not contribute to the nonlinear phase shift (when a 50/50 coupler is used), the transmission still shows up strong intensity-dependent behaviour. Note that the purpose of the strong twist applied to the fibre is to reduce the impact of the fibre linear birefringence, which is not controlled and may vary with environmental conditions, thus being responsible of unstable NOLM behaviour. By taking over

Fig. 2. General scheme of the NOLM structure investigated throughout this paper. QWP: quarter-wave plate retarder. \( L_1, L_2 \): transmissions of tuneable attenuators (\( L_1 = L_2 = 1 \) if no loss is introduced).
the linear birefringence, the circular birefringence imposed by twist (optical activity) will improve noticeably the robustness of the device, as it will be shown later in this paper. Passive mode locking of a figure-eight laser including such a NOLM (with a 50/50 coupler) was successfully demonstrated experimentally [14]. Note that other designs of NOLMs including a symmetrical coupler are possible, in particular schemes using highly birefringent fibre in the loop [2,15].

The interferometer under study is shown in Fig. 3. It consists in an optical coupler whose output ports 3 and 4 are connected using a section of low-birefringence, highly twisted fibre. A QWP is inserted in the loop at port 4. Light is launched into the NOLM through port 1 of the coupler, and the transmitted beam is collected at port 2. In order to determine the transmission of the NOLM for a given input polarisation, we must first analyse the evolution of both the polarisation state and the phase of the fields that counter-propagate in the loop. Let us first consider the linear case, i.e., when light propagation is governed by the fibre birefringence only. This case can be advantageously investigated using a transfer matrix approach. As we are considering highly twisted, low-linear-birefringence fibre, we may neglect the linear birefringence with respect to the strong twist-induced circular birefringence. In these conditions, the transmission matrix of the fibre, expressed in the circular $[C^+, C^-]$ polarisation basis, is given simply by

$$F = \begin{bmatrix} e^{i\beta} & 0 \\ 0 & e^{-i\beta} \end{bmatrix}, \quad \beta = \frac{hqL}{2n},$$

where $q$ is the fibre twist rate (in radians per unit length), $h \approx 0.13\sim0.16$ for silica fibre, $L$ is the fibre length and $n$ its refractive index. The effect of the fibre is thus simply to rotate by an angle $\beta$ the polarisation state of the input light. The transfer matrix of the QWP can also be easily calculated. Its angle $\alpha$ is defined as shown in Fig. 3. We thus obtain, in the $[C^+, C^-]$ basis, for clockwise and counter-clockwise beams, respectively [16],

$$\text{QWP}_{cw} = \begin{bmatrix} \frac{1 + i}{2} & \frac{1 - i}{2} e^{2i\alpha} \\ \frac{1 - i}{2} e^{-2i\alpha} & \frac{1 + i}{2} \end{bmatrix},$$

$$\text{QWP}_{ccw} = \begin{bmatrix} \frac{1 + i}{2} & \frac{1 - i}{2} e^{-2i\alpha} \\ \frac{1 - i}{2} e^{2i\alpha} & \frac{1 + i}{2} \end{bmatrix}.$$

Note that $\text{QWP}_{ccw}$ can be deduced from $\text{QWP}_{cw}$ simply by replacing $\alpha$ by $-\alpha$. Finally, if $r$ is the power coupling ratio of the coupler ($0 < r < 1$), the electric field is multiplied by $\sqrt{r}$ after reflection through the coupler (from 1 to 3 or from 4 to 2), whereas it is multiplied by $i\sqrt{(1 - r)}$ after transmission (from 1 to 4 or from 3 to 2). Considering for example a circular-right input polarisation $E_{in} = [1; 0]$ at port 1 of the coupler, one can calculate the output field at port 2 after clockwise and counter-clockwise propagation through the loop, respectively:
The total output field is given by the sum of both fields. It appears from the above expressions that circular-right components do not depend on the QWP angle \( \alpha \), and that they are in phase opposition, so that the transmission for this polarisation component is minimum, independently of the QWP position. In contrast, the phase of each circular-left component depends on the QWP angle. In the important particular case of the symmetrical coupler \( r = 0.5 \), Eq. (3) lead to:

\[
E_{\text{out}} = E_{\text{out},cw} + E_{\text{out},ccw} = \begin{bmatrix}
0 \\
\frac{1+i}{2} \sin(\beta - \alpha)
\end{bmatrix}.
\]

(4)

The circular-right polarisation components thus cancel out in all cases in the linear regime, so that the light transmitted by the NOLM is a purely circular-left polarised beam, whose amplitude depends sinusoidally on \( \alpha \). Defining the power transmissions \( T_{tr} \), as the ratio between the output power in the circular-right state and the input power (in that same polarisation state), and \( T_{tl} \) as the ratio between the output power in the circular-left state and the input power, we have:

\[
T_{tr} = 0;
T_{tl} = \frac{1}{4}[1 - \cos(2\beta - 4\alpha)].
\]

(5)

The fraction of the input power transmitted into the circular-left polarisation state thus oscillates between 0 and 0.5, with a period of \( \pi/2 \) in \( \alpha \). In the general case \( r \neq 0.5 \), we have:

\[
T_{tr} = \frac{(1 - 2r)^2}{2};
\]

\[
T_{tl} = \frac{(1 - 2r)^2}{2} + r(1 - r)[1 - \cos(2\beta - 4\alpha)].
\]

(6)

The transmitted circular-right polarisation component is still minimum, although different from zero for \( r \neq 0.5 \). The transmission of the circular-left polarisation oscillates between this minimum value and 0.5, again with a period of \( \pi/2 \). Note also that the values of \( \alpha \) for which \( T_{tl} \) is minimum (maximum) do not depend on the coupling ratio \( r \).

3. Weakly nonlinear regime

Let us now investigate how the NOLM transmission is modified in the nonlinear regime. The nonlinear polarisation evolution (including effects of SPM and XPM) of an optical beam along a fiber can be easily described using the approximate, weak-nonlinearity equations of motion, which were developed by Kuzin et al. [13] on the basis of the general coupled equations for nonlinear polarisation evolution in the continuous-wave case [17]. This set of two differential equations describes the evolution along the fiber of the two polarisation components, which are expressed in the basis of the elliptical polarisation eigenmodes, \([S^+, S^-]\). In the case of a highly-twisted fibre however, we may consider that the elliptical eigenmodes coincide with the circular polarisation states, so that these equations are

\[
i\partial_t C^+ = \left[ -\mu - \frac{3}{2} P_N + \frac{1}{4} P_N \sin^2 \Phi_0 
+ \frac{1}{2} P_N A_S \left( \cos^2 \Phi_0 - \frac{1}{2} \sin^2 \Phi_0 \right) \right] C^+;
\]

\[
i\partial_t C^- = \left[ -\mu - \frac{3}{2} P_N + \frac{1}{4} P_N \sin^2 \Phi_0 
- \frac{1}{2} P_N A_S \left( \cos^2 \Phi_0 - \frac{1}{2} \sin^2 \Phi_0 \right) \right] C^-.
\]

(7)

These equations are written in a rotating basis, which twists along with the fibre, at a rate corresponding to the twist rate, \( q \). In Eq. (7), \( \mu = \sqrt{(\eta^2 + g^2)} \), where \( g = \gamma \pi/k \) is the ratio of circular to linear birefringence, as \( k = \pi \delta n/\lambda \) describes the linear birefringence, and \( \gamma = \left[ h/(2n) - 1 \right] q \) the circular birefringence, in the rotating frame. \( P_N = b \pi P_n/k \) is the normalised power, where \( P_n \) is the input power (into the fibre) and \( b = 4\pi n_2/3\lambda A_{eff} \) is
the nonlinearity ($A_{\text{eff}}$ being the effective modal area, and $n_2$ the Kerr coefficient). The derivatives are taken with respect to a normalized length $s = zk/\pi = z/L_b$, where $z$ is the absolute length (in m) and $L_b$ the fibre beat length. Finally, $A_s = |S^+|^2 - |S^-|^2 \approx |C^+|^2 - |C^-|^2$ is the Stokes parameter and $\Phi_0 = \text{atan}(\pi/g) \approx 0$ in the case of strong twist.

Let us consider again that a right-circularly polarised beam is injected into the highly-twisted NOLM through port 1 of the coupler. Having a look at Fig. 3, it appears that a fraction $r$ of this light will be launched into the fibre through port 3 of the coupler without change in its polarisation state. Moreover, Eq. (7) show that, in the weak nonlinearity approximation, this initial polarisation state is not modified during propagation by the nonlinear terms in $P_N$. From this point of view, light behaves just like in the linear case, the only difference being that a nonlinear phase shift accumulates during propagation. As a result, after clockwise propagation in the loop, and before crossing the QWP, light still maintains its initial circular-right polarisation. The QWP then turns this circular-right polarisation into linear polarisation, so that the clockwise beam reaches port 4 in this polarisation state. Let us now consider the counter-clockwise beam. It corresponds to that fraction of the input beam that is coupled from port 1 to port 4. Again, the coupler does not affect the polarisation state, so that right-circularly polarised light crosses the QWP before being launched into the fibre. The QWP turns this right-circular polarisation into linear polarisation, i.e., the sum of a circular-right and a circular-left polarisation components, with equal powers, which will propagate together along the fibre. Again, Eq. (7) show that, during propagation, no power transfer will occur between these two components, the nonlinear terms affecting only their phases independently. This means that the polarisation state of the counter-clockwise beam will be kept linear until it reaches port 3. In summary, the propagation of the clockwise beam is simply described by

$$\begin{align*}
    i\partial_s C^+ &= \left[-\frac{A_s}{2} r P_N + \frac{1}{2} r P_N \right] C^+ \\
    &= \left[-\mu - r P_N \right] C^+,
\end{align*}$$

where we used that $\Phi_0 = 0$, $A_s = 1$ for circular-right polarisation, and $P_N = r P_{IN}$ the fraction of the input power $P_{IN}$ propagating in the fiber in the clockwise direction. On the other hand, propagation in the counter-clockwise direction is governed by

$$i\partial_s C^\pm = \left[-\frac{3}{2} \left(1 - r\right) r P_{IN} \right] C^\pm,$$

where we used again that $\Phi_0 = 0$, $A_s = 0$ in this case for linear polarisation, and the fraction of the input power $P_{IN}$ that propagates in the counter-clockwise direction is $P_N = \left(1 - r\right) P_{IN}$.

The transmission of the polarisation components, circular-right and circular-left, through the NOLM, is a function of the difference between the phase shifts experienced by the counterpropagating beams. In the case of the clockwise signal, the nonlinear phase shift undergone by the sole circular-right component traveling along the fibre (see Eq. (8)) will be transmitted to both circular-left and circular-right components that will be generated by the QWP. In the case of the counter-clockwise signal, the circular-left and circular-right components evolve independently along the fibre, but both experience the same nonlinear phase shift, as it can be seen from Eq. (9) (the nonlinear term does not depend on the sign of the circular polarisation). In conclusion, if $\Delta \phi_0$ is the linear phase difference between counterpropagating circular-right (circular-left) polarisation components, the nonlinear phase difference, $\Delta \phi_{nl} = \Delta \phi - \Delta \phi_0$, is identical for both circular-right and circular-left polarisations. In contrast, the linear phase difference $\Delta \phi_0$ is generally different for circular-right and circular-left polarisations, as it appears from Eq. (3) (we find indeed that $\Delta \phi_{0r} = \pi$ and $\Delta \phi_{0l} = \pi + 2\beta - 4\alpha$ for circular-right and circular-left polarisations, respectively). Note that $\Delta \phi_0$ is determined by the linear phase shifts due to the fibre, as well as to the other elements constituting the NOLM (coupler and QWP). Let us now calculate the nonlinear phase difference $\Delta \phi_{nl}$. Integrating Eqs. (8) and (9) along the fibre length, it comes that

$$\Delta \phi_{nl} = \Delta \phi - \Delta \phi_0 = \frac{5}{2} \left(1 - \frac{3}{5} \right) P_{IN} l,
$$

where $\mu = -\frac{A_s}{2}$.
where \( l = L/L_b \) is the normalised fibre length. As expected, this expression is identical for both circular polarisation states. This phase difference is proportional to the input power, and is responsible for a sinusoidal dependence of the power transmission for both polarisations, \( T_{rr} \) and \( T_{rl} \), in function of input power. We will now calculate the critical power \( P_c \), i.e., the power needed to switch the NOLM from minimum to maximum transmission, or conversely. This parameter can be readily obtained from Eq. (10), taking the nonlinear phase difference equal to \( \pi \). This leads to

\[
P_c = \frac{2\pi}{5} \frac{1}{l(r - \frac{1}{2})}.
\]

Again, this relation is valid for both \( T_{rr} \) and \( T_{rl} \). Eq. (11) shows that the critical power strongly depends on the coupling ratio \( r \), and can even tend to infinity in the particular case \( r = 0.6 \). Note that, for \( r < 0.6, P_c < 0 \). Although it is clear that physically, the critical power should always be positive, and that strictly speaking we should take the modules of the right-hand side in Eq. (11), we keep this definition as it is in order to avoid carrying \( +/\) signs in the expression of \( \Delta \phi_{nl} \). In Fig. 4 are presented, for some values of \( r \), typical evolutions of \( T_{rr} \) and \( T_{rl} \) in function of both the QWP angle \( \alpha \) and the normalised input power \( P_{IN} \). For these plots, the polarisation evolution along the fibre was obtained from the numerical resolution of the general coupled equations for nonlinear polarisation evolution in the continuous-wave case [17]. These results are found to be in very good agreement with our analysis that used the matrix approach and the weak nonlinearity model.

In summary, considering a circular-right input polarisation, the nonlinear transmission characteristics for the circular-right (\( T_{rr} \)) and circular-left (\( T_{rl} \)) components can be expressed as

\[
T_{rr/rl} = \frac{(1 - 2r)^2}{2} + r(1 - r) \\
\times \left[ 1 - \cos \left( \Delta \phi_{nl} + \Delta \phi_{0,rr/rl} - \pi \right) \right],
\]

where the nonlinear phase difference \( \Delta \phi_{nl} = \pi P_{IN}/P_c \) is identical for both polarisation states, whereas the linear phase difference \( \Delta \phi_{0,rr} = \pi \) for circular-right polarisation and \( \Delta \phi_{0,rl} = \pi + 2\beta - 4\alpha \) for circular-left polarisation. Note that these values of the linear phase differences were obtained using the matrix approach, however the same results can also be obtained by using Eqs. (8) and (9) for the fibre contribution, taking \( P_{IN} = 0 \) (the QWP contribution is still obtained through matrixes). Eq. (12) and Fig. 4 shows that, whatever the angular position of the QWP, the nonlinear transmission function of the circular-right component basically always behaves in the same way: starting from minimum transmission for \( P_{IN} = 0 (\Delta \phi_{nl} = 0) \), it then increases for increasing input power, reaching a maximum of 0.5 for \( P_{IN} = |P_c| \), and decreases again for \( P_{IN} > |P_c| \). In contrast, the nonlinear characteristic of the circular-left component strongly depends on the QWP angle, as it defines the bias of the sinusoidal transmission function (its period, \( 2/P_c \), however, is fixed by the coupling ratio \( r \) and thus remains constant as \( \alpha \) is changed). The effect of rotating the QWP is thus to translate this transmission characteristic along the input power axis. The limit case \( r = 0.6 \) must be interpreted by considering that the critical power \( |P_c| = \infty \), so that the dependence of the transmissions on input power vanishes (in particular, \( T_{rr} \) remains minimum over the whole plane). The observation of the dependence on \( \alpha \) of the circular-left transmission \( T_{rl} \) inspires an easy way to control the position of the NOLM maximum transmission, which can be tuned continuously simply by tuning the angle of the QWP. Provided that the coupling ratio \( r \), which defines the critical power, is properly chosen, the tuning range of the maximum transmission is wide enough for practical applications (like amplitude regularisation), while keeping the low-power transmission sufficiently low. Although all these developments were made for a circular-right input polarisation only, it has to be mentioned that the NOLM behaviour is similar, mutatis mutandis, in the case of a circular-left input polarisation. Note finally that the use of the \( T_{rl} \) transmission characteristic implies the selection of the circular-left component at the NOLM output (this can be done using a QWP followed by a polariser whose axis is set at 45° with respect to the QWP axes).
Fig. 4. Plots of the transmissions $T_{rr}$ (a, c, e, g) and $T_{rl}$ (b, d, f, h) in function of QWP angle $\alpha$ and normalised input power $P_{IN}$, for circular-right input polarisation. The parameters of the simulation are: $g = 200$, $l = 10$ and $r = 0.4$ (a, b), 0.5 (c, d), 0.55 (e, f) and 0.6 (g, h). We also took $n = 1.45$ and $h = 0.15$. Note that the chosen value of $g$, corresponding to $\theta_0 = \arctan(\pi/200) = 0.015$ rad, is realistic (compatible with experimental values of $q \approx 3$ turns/m and $L_0 \approx 10$–20 m). For such parameters, and considering standard single-mode fibre, $P_{IN} = 1$ represents about 50–100 W at the NOLM input.
4. Tuneable NOLM with invariant output polarisation

Although the method described above provides a versatile way to tune the maximum transmission of the NOLM, it suffers from some drawbacks. Firstly, due to the selection of only one circular polarisation component at the NOLM output, the maximum transmission is limited to 0.5. Moreover, when the position of this maximum is tuned away from \([P_N]\), the minimum transmission no longer appears exactly at \(P_{IN} = 0\), so that the acceptable tuning range is reduced when a pedestal suppression effect is desired, if the input signal is a pulse train (see Fig. 1). In order to solve these problems, let us first tune the QWP angle until \(T_{r1}\) reaches its minimum value for \(P_{IN} = 0\). This condition is met when \(\alpha = \beta/2 + k\pi/2, k\) integer. As the right-circular transmission \(T_{rr}\) is always minimum at low power, we have that \(\Delta \phi_{0,rr} = \Delta \phi_{0,rl} = \pi\) at this angular position, and the global NOLM transmission \(T = T_{r1} + T_{rl}\) (including both polarisation components) is ensured to be minimum for \(P_{IN} = 0\) \((T_{min} = 0\) if \(r = 0.5\)). Moreover, as, for each value of the input power, \(\Delta \phi_{0,l}\) is the same for both circular polarisation components, the total phase differences \(\Delta \phi_{0,rr} = \Delta \phi_{0,rl}\), and thereby the magnitudes of both output polarisation components (or equivalently the power transmissions \(T_{rr}\) and \(T_{rl}\), see Eq. (11)) are equal functions of the input power. As the sum of equal-amplitude circular-left and circular-right polarisations yields a linear polarisation, the NOLM output is in this case linear for any value of the input power. In particular, for \(P_{IN} = |P_c|\), \(T_{rr} = T_{rl} = 0.5\) so that \(T_{max} = T_{rr} + T_{rl} = 1\): the contrast between maximum and minimum transmission \(T_{min} = (1 - 2r)^2\) is maximal. Finally, using the weak nonlinearity analysis, it can be shown that the angle of the output linear polarisation \(\alpha = 0.5[\mu l + g l/(h/(2n) - 1)] \pm \pi/4\), so that it does not depend on the input power: for a circular-right input polarisation, the polarisation state of the NOLM output is thus essentially linear and invariant. This interesting property would make it easy to couple the transmitted beam into a polarisation-maintaining fibre, for example. Fig. 5(a) and (b) shows the power transmission \(T\), as well as the output polarisation states for different input powers, in the case \(\alpha = \beta/2\). The exact nonlinear propagation model [17] was used. The figure clearly confirms our conclusions. Note that, by symmetry, a similar behaviour of the NOLM is found using a circular-left input polarisation.

In the above, by fixing the QWP angle \(\alpha\) to a particular value, we ensured minimum transmission at low power, maximal contrast between maximum and minimum transmission, as well as a constant and linear output polarisation state (for a circular-right input). However, this also suppresses all possibility to tune the position of maximum transmission by way of \(\alpha\). To tune this maximum position while keeping minimum transmission for \(P_{IN} = 0\), the only way is to tune the critical power, \(P_c\). As it appears clearly from Eq. (11), this can be done, in theory, by adjusting the coupling ratio \(r\) of the coupler. In fact, the value of \(r\) determines the power asymmetry between counter-propagating beams and, thereby, the nonlinear phase difference between them and the period of the nonlinear transmission characteristic. Unfortunately, in most cases, this value of \(r\) is intrinsic of the coupler that was used in the NOLM design, and can hardly be tuned in practice. Nevertheless, it is possible, by inserting a tuneable loss after one of the output ports of the coupler, to generate a power asymmetry between the counter-propagating beams, which will operate many in the same way as a tuneable coupling ratio (Fig. 2). As an example, let us consider the case of a NOLM including a symmetric (50/50) coupler and a tuneable loss (transmission \(L_2 < 1\)) inserted at port 4 of the coupler (Fig. 2, with \(L_1 = 1\)). Because this loss reduces the power of the counter-clockwise beam in comparison with the clockwise beam, its effect is similar to a value of \(r > 0.5\). Using the weak nonlinearity model, it is easy to show that the critical power obeys the law:

\[
P_c = \frac{2\pi}{l[2L_1 + 3L_2] - 3L_2].
\]

For \(r = 0.5\) and \(L_1 = 1\), Eq. (13) simplifies as

\[
P_c = \frac{4\pi}{l(2 - 3L_2)}.
\]

Eq. (14) shows that, by tuning \(L_2\) between 1 and 2/3 it is possible to adjust \(|P_c|\) to any value between
$p = l$ and infinity. The limit case is obtained for $L_2 = 2$, i.e., when the power ratio between counter-propagating beams is $3/2$. This case is similar to $r = 0.6$ when there is no loss (in this case, the power ratio $= 0.6/0.4 = 3/2$, too).

In the case of a lossless NOLM, we have seen that the linear phase difference between counter-propagating beams does not depend on the coupling ratio $r$ ($\Delta \phi_{01} = \pi$ in all cases for circular-right polarisation and $\Delta \phi_{01} = \pi + 2\beta - 4\alpha$ only depends on twist and QWP angle for circular-left polarisation). By analogy, this remains valid when we consider a loss $L_2$ in the NOLM with symmetric coupler. As a consequence, if $a$ was adjusted so as to bias the minimum transmission at $P_{IN} = 0$, the adjustment of $L_2$ does not modify this setting. More than this, whereas a value of $r \neq 0.5$ increases this minimum transmission from 0 (see Eq. (12)), the minimum transmission is maintained to zero if $L_2 < 1$ in a NOLM with symmetrical coupler. Indeed, as both counter-propagating beams have crossed once $L_2$ at the time they interfere at the coupler, they have equal magnitudes and will cancel out completely if their phases are opposed. This complete cancellation of low-power transmission is a crucial advantage in particular for optimal pedestal suppression. The equality of losses undergone by counter-propagating beams also ensures that, if

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**Fig. 5.** Nonlinear transmission characteristic $T$ (a) and corresponding output polarisation evolution (b) of the NOLM with $a = \beta/2 = 2.0$ rad and $r = 0.5$. The other parameters are the same as in Fig. 4(c) and (d), which leads to $a = 2.9$ rad. The coordinate frame $[x, y]$ corresponds to the $[x_1, y]$ and $[x_2, y]$ systems shown in Fig. 3 (which are equivalent at the NOLM output). When the input power increases, the polarisation state at the NOLM output remains linear along this direction, and only the magnitude of the output field is modified. In contrast, for other values of $a$, the transmission characteristic departs from the optimal curve (c) and the output polarisation state is not linear and changes with power (d) (curves obtained for $a = 1.8$ rad).
the angle $a$ is properly adjusted as explained before, the constant linear polarisation condition will be maintained at the NOLM output. It has also to be stressed that these interesting properties of the NOLM (high contrast and constant linear output polarisation) are preserved if undesirable losses are present in the loop, like in particular the insertion loss of the QWP. The NOLM properties will depart from the ideal case only if the QWP shows up some polarisation-dependent loss. In practice however, a QWP can be readily obtained by wrapping the fibre a given number of turns around a circle of properly chosen diameter, which avoids introducing insertion loss or polarisation-dependent loss.

There are two disadvantages, however, of this loss-tuning approach with respect to the unpractical tuning of the coupling ratio. First, considering the same power ratio between counter-propagating beams, loss tuning leads to a higher critical power, which means that higher input power is required to obtain switching. This can be verified using Eqs. (11) and (14) and introducing the power ratio $Q = r/(1 - r)$ for coupling ratio tuning and $Q = L_1/L_2 = 1/L_2$ for loss tuning, and remembering that $Q > 1$ ($r > 0.5$ or $L_2 < 1$) in our case. This difference of critical power is illustrated in Fig. 6, and is easily explained by the lower power of both beams in the loss-tuning approach (for the same value of $Q$). Another drawback of the loss-tuning approach is that the value of the maximum transmission is reduced from 1 to the value of $L_2$ (see Fig. 6). The impact of these drawbacks can be limited however by a proper choice of the NOLM construction parameters, like the coupling ratio and the fibre length (which both define the loss-free value of $P_c$). Indeed, if these parameters are well chosen considering the expected average (peak) power of the input signal, the loss excursion needed to optimise the NOLM operation will be minimal. Note that, if necessary, these drawbacks can be eliminated by inserting a tuneable gain at port 3 instead of a tuneable loss at port 4 (although more expensive, this solution includes the possibility of reducing the critical power and increasing the maximum transmission above 1).

From these results, we infer a simple and reproducible adjustment procedure that will lead in all cases to the optimal NOLM configuration for amplitude regularisation and pedestal suppression of an optical pulse train. We consider here again that the input polarisation state is circular-right (or left). The first step of the adjustment is made at low input power (a continuous-wave source may be used in this case). It consists in tuning the QWP angle until the optical power measured at the NOLM output is minimum. A rotation of less than a quarter of a turn is required. When this position is reached, the condition $a = b = 2 + k\pi/2$ is necessarily fulfilled. For the second step, the pulse train is coupled into the NOLM, and the NOLM output is monitored, for example on an oscilloscope. If step 1 has been performed correctly, the pedestals that were possibly accompanying the pulses should have disappeared after transmission through the NOLM. The loss $L_2$ is then tuned so as to minimise the amplitude fluctuations of the pulses. Finally, if the output signal is to be injected into a polarisation-sensitive system (polarisation-maintaining fibre, electro-optic modulator,...), the angle $a$ of the linear output polarisation should be known with precision. This angle can be easily measured by inserting a rotating polariser between the NOLM output and a power meter. It corresponds to the

![Fig. 6. Critical power and maximum NOLM transmission in function of the power ratio $Q$ between counter-propagating beams, for both the coupling-ratio-tuning (dashed line) and loss-tuning (solid line) approaches. Data for $Q < 1$ in the case of the loss-tuning approach were obtained by considering in Fig. 2 that $L_2 = 1$ and $L_1 < 1$ (2/3 $< L_1 < 1$ for 2/3 $< Q < 1$). The figure shows however that tuning $L_1$ is not interesting, as a substantial reduction in $T_{\text{max}}$ is associated with a very small variation of the critical power.](image)
angle of the polariser associated with maximum output power. Note that, at 90° from this angle, the power detected through the polariser should be close to zero if the polarisation is close to linear: this property may be used to verify that the first step of the adjustment procedure was correctly achieved. Moreover, this may refine the measured value of $a$, as the angle for minimum transmission is likely to be measured with higher precision than the angle at which the transmission through the polariser is maximum.

As mentioned in the beginning of this paper, the birefringence of a low-birefringence fibre may fluctuate, as a result of environmental perturbations. An important issue is thus the sensitivity of the NOLM properties to small changes in the amount of linear birefringence. Fig. 7 shows the NOLM transmission $T$ as a function of input power, in the same conditions as in Fig. 5(a), except that the beat length of the twisted fibre is varied around $L_b = L/10$ (the fibre length $L$ is kept constant). The figure shows that, when the birefringence is varied over a range corresponding to a difference as large as two beat lengths along the fibre, the transmission characteristic remains essentially unchanged. We also verified that the output polarisation was kept linear, with the same angle $a$ for all values of $L_b$. These results show that the NOLM with highly-twisted fibre is robust to uncontrolled modifications of the fibre birefringence, which may occur typically as a consequence of temperature changes or mechanical vibrations.

It has to be noted that all these results are valid, strictly speaking, for a circular input polarisation (right or left). If the input polarisation state fluctuates significantly, some deviation from the optimal NOLM behaviour is to be expected, so that the input polarisation should be kept stable in practice for proper NOLM operation. Another interesting point is the behaviour of the NOLM for input polarisation other than circular. Although the theoretical analysis is complicated and no longer allows such simple interpretation as in the circular case, numerical simulations allowed observing similar NOLM operation for various input polarisation states. In particular, considering linear input polarisation, it appeared that, for some values $x$ of the QWP angle, separated by $\pi/2$, the complete switch of the NOLM (from $T = 0$ to 1) is also reached, with a roughly constant output polarisation state, which is in this case circular (left or right, depending on the value of $x$).

Another important fibre-based amplitude regularisation method was demonstrated by several authors. Indeed, thanks to the effect of nonlinear polarisation rotation in a section of fibre followed by a polariser, a nonlinear transmission characteristic can be generated. This method, called additive pulse limiting (APL), also allows the adjustment of both the phase and the period of the transmission characteristic, and was demonstrated for supermode noise reduction [18,19] and pulse amplitude equalisation in a rational harmonically modelocked fibre laser [20]. However, as this adjustment procedure relies on PCs, the optimal nonlinear dependence can only be found empirically, and the reproducibility of this procedure is not guaranteed. In contrast, there is in our approach a well-defined relationship between the NOLM parameters (QWP angle $x$ and loss $L_2$) and the transmission characteristic, so that we were able to propose a perfectly reproducible adjustment procedure. Moreover, contrary to our method, APL is sensitive to birefringence fluctuations, so that the adjustment procedure should be repeated periodically for optimal amplitude limiting operation.
5. Conclusion

In conclusion, we studied theoretically the behaviour of a fibre Sagnac interferometer including low-birefringence, highly twisted fibre in the loop, together with a QWP. Using simple analytical models, we were able to infer the important properties of the interferometer in both the linear and the weakly nonlinear regimes. In particular, we have shown that, considering a circular input polarisation, the transmission characteristic of the orthogonal polarisation component can be shifted continuously through rotation of the QWP. This constitutes an easy way to adjust the position of the maximum transmission of the interferometer, and could be used for applications like the reduction of supermode beat noise in harmonically mode-locked fibre lasers, or damping of relaxation oscillations. This device would also be very helpful in the case of rational harmonically mode-locked or subharmonic synchronous mode-locked lasers, which are able to generate pulse trains at very high repetition rates, but can generally not avoid the presence of a parasitic amplitude modulation at a subharmonic of the repetition rate. We have also shown that, for particular wave plate angles, the contrast between maximum and minimum transmission is maximal, whereas the output polarisation is constant and independent of the input power. In this case, optimal amplitude regularisation can be obtained by tuning a loss inserted in the cavity, at the expense of a slight increase in the interferometer insertion loss. Moreover, as the minimum of transmission appears in the low input power region, optimal pedestal suppression is guaranteed, in addition to a pulse shortening effect. Finally, we have demonstrated that the interferometer is robust to variations of the fibre linear birefringence, which may originate from environmental perturbations. As a consequence, this device looks very promising for a wide range of applications requiring a robust and easily adjustable intensity-dependent transmission characteristic with invariant output polarisation.

Acknowledgements

O. Pottiez is supported by CONACyT (Mexican Council for Science and Technology) and by F.N.R.S. (Belgian Fund for Scientific Research). This work was also funded by CONACyT Project J36135-A.

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