Supercontinuum generation in optical fibers: 40 years of nonlinear optics in one experiment

Goëry Genty
Optics Laboratory, Tampere University of Technology, Finland
Part I: back to basics

- Optical fibers for supercontinuum generation
- Ultrafast pulse propagation and supercontinuum generation
- Visualizing supercontinuum dynamics
- Coherence properties
Part II: recent developments

- New fibers, new materials and new spectral regions
- Supercontinuum instabilities & optical rogue waves
- Bridging the gap between traditional nonlinear optics and supercontinuum generation
- Future steps and challenges
Combined stimulated Raman scattering and continuum self-phase modulations

Joel I. Gersten
Institute for Advanced Studies, Hebrew University, Mount Scopus Campus, Jerusalem, Israel

R. R. Alfano and Milivoj Belić
Department of Physics, The City College of New York, New York, N. Y. 10031
(Received 8 January 1979)

A theory describing the combined effects of stimulated Raman scattering and continuum self-phase modulation is developed. As may be expected, the effects are not simply additive. Calculations are presented which determine the interaction of these effects in various limits.

In the case of self-phase modulation (SPM), however, the repopulation of the spectral intensity takes place in a more gradual manner. Owing to the nonlinearity of the medium, the pulse heterodynes against itself and gradually increases its spectral width. There is a continuum of frequencies produced in this process. Supercontinuum generation spanning the visible and infrared region was first observed by Alfano and Shapiro when intense picosecond laser pulses were passed through liquids and solids.\footnote{\textsuperscript{3}}
Supercontinuum timeline

1970
- Nonlinearity in silica fiber (R. Stolen & Lin)

1980
- Experimental Fiber solitons (L. Mollenauer)
- Femtosecond Ti:Sapphire (Sibbett)

1990
- Supercontinuum generation in bulk silica (R. Alfano)
- Supercontinuum generation in photonic crystal fiber (J. Ranka et al.)
- Femtosecond Ti:Sapphire (Sibbett)

2000
- Solid core photonic crystal fiber (P. Russell)
- Nobel prize Hall & Hänsch

2005
- Photonics@be

Today

Supercontinuum generation in silica fiber (R. Stolen & Lin)
Why the excitement?
Supercontinuum has been around for decades

Originally in bulk

Nonlinear medium

Picosecond laser

Supercontinuum

Volume 24, Number 11

PHYSICAL REVIEW LETTERS
16 March 1970

OBSERVATION OF SELF-PHASE MODULATION AND SMALL-SCALE FILAMENTS IN CRYSTALS AND GLASSES

R. R. Alfano* and S. L. Shapiro
Bayside Research Center of General Telephone & Electronics Laboratories Incorporated,
Bayside, New York 11360
(Received 10 December 1969)

EMISSION IN THE REGION 4000 TO 7000 Å VIA FOUR-PHOTON COUPLING IN GLASS

R. R. Alfano and S. L. Shapiro
Bayside Research Center of General Telephone & Electronics Laboratories Incorporated,
Bayside, New York 11360
(Received 9 January 1970)
There are some issues though

- Limitations
  - Walk off
  - Diffraction
  - Strong dispersion, limited broadening

- Need very high energy
  - Damage
Optical fibers are more convenient

- Pulse propagation in optical fibers
  - No diffraction, long interaction length

- Dispersion/nonlinearity can be controlled: crucial!
  - propagation dynamics depend on pump wavelength relative to fiber zero dispersion wavelength (ZDW)

“NEW” FIBERS
- Small core, high doping
- Photonic crystal fibers
- Tapered fibers
- Non-silica materials

PARAMETER CONTROL
- Dispersion
- nonlinearity
- Confinement

APPLICATIONS
- Supercontinuum
- Frequency conversion
- Pulse compression
- Regeneration
- Amplification
Optical fibers for nonlinear applications

- Chemistry and geometry both contribute to the refractive index profile
  - Determine modal confinement (nonlinearity) and dispersion characteristics
  - Wide range of possibilities

![Diagram showing different types of fibers: Single Mode Fiber, Highly Nonlinear Fiber, Photonic Crystal Fiber, Taper / Microfiber. The diagram illustrates the core regions with different materials and cross-sectional areas, indicating large area for weakly nonlinear and small area for highly nonlinear.]
Photonic crystal fibers

- Generally single material with a high air-fill fraction

- ZDW displaced to shorter wavelengths: wavelength of high-power short pulse sources

- Large nonlinearity (x100 compared to standard fibers) ⇒ nonlinear effects dramatically enhanced
Taper/microfibers

- Tapering standard fibers: similar properties to photonic crystal fibers
Key aspects for supercontinuum

Micro/nano-structured waveguides: engineered nonlinearity and dispersion

- Dispersion shifts soliton regime to shorter wavelengths
- Long interaction lengths
- Light tightly confined → very large intensities!
Pulse propagation in optical fibers

\[ E(r, z, t) = F(r)A(z, t)e^{i\omega_0 t} \]

- Modal distribution does not vary with propagation
- Consider only the fundamental mode LP01 (Gaussian distribution)
- Only temporal effects matters, no diffraction
Nonlinear pulse propagation in optical fibers

- Pulse propagation in optical fiber obeys:
  \[
  \nabla \times \nabla \times E(r,t) + \frac{1}{\varepsilon^2} \frac{\partial^2 E(r,t)}{\partial t^2} = -\mu_0 \frac{\partial^2 P(r,t)}{\partial t^2}
  \]

  with \( P(r,t) = (P_L(r,t) + P_{NL}(r,t)) \).

\[
P(r,t) = \varepsilon_0 \chi^{(0)} + \varepsilon_0 \chi^{(1)} E(r,t) + \varepsilon_0 \chi^{(2)} E^2(r,t) + \varepsilon_0 \chi^{(3)} E^3(r,t) + \ldots
\]

- In silica: \( \chi^{(2)} = 0 \) (centro-symmetric material)

- Only THIRD-ORDER nonlinear effects
The Kerr effect

- Light intensity changes the refractive index:
  \[ n = n_0 + n_2 I = n_0 + \left( \frac{n_2}{A_{\text{eff}}} \right) P \]
  - \( n_2 \) proportional to \( \chi^{(3)} \): nonlinear refractive index
  - \( A_{\text{eff}} \) effective area of the fiber mode

- Important parameter: nonlinear coefficient \( \gamma = \frac{n_2 \omega}{c A_{\text{eff}}} \)
  - Nonlinear effects scales with \( \gamma \)

- In silica, \( n_2 \approx 3 \times 10^{-20} \text{ m}^2/\text{W} \): weak nonlinear material! But silica has the lowest losses…

- Need small \( A_{\text{eff}} \)
Nonlinear effects in optical fibers

- Pump wavelength is crucial!

Diagram showing the relationship between dispersion, wavelength, and nonlinear effects such as self-phase modulation, four-wave mixing, solitons, and modulation instability. The diagram indicates that certain conditions lead to broadest spectra.
Modeling pulse propagation

- **Generalized nonlinear Schrödinger Equation (GNLSE)**

\[
\frac{\partial A}{\partial z} + \frac{\alpha}{2} A - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left(1 + i\tau_{\text{shock}} \frac{\partial}{\partial T}\right) \left(A(z, t) \int_{-\infty}^{+\infty} R(T') |A(z, T - T')|^2 dT'\right)
\]

- **Loss**
- **Dispersion**
- **Self-Steepening term**
- **SPM, FWM, Raman**

**Intensity-dependent ref. index**

\[
\gamma = \frac{n_2 \omega_0}{c A_{\text{eff}}}
\]

**Raman response**

\[
R(T) = (1 - f_R)\delta(T) + f_R h_R(T)
\]

- Can include noise, frequency-dependent mode area, polarization...etc

- Validity: down to single cycle regime
Generalized nonlinear Schrödinger Equation (GNLSE)

\[ \frac{\partial A}{\partial z} + \frac{\alpha}{2} A - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i\gamma \left( 1 + i\tau_{\text{shock}} \frac{\partial}{\partial T} \right) \left( A(z, t) \int_{-\infty}^{+\infty} R(T')|A(z, T - T')|^2 dT' \right) \]

- Loss
- Dispersion
- Self-Steepening term
- SPM, FWM, Raman

Intensity-dependent ref. index
\[ \gamma = n_2\omega_0/c A_{\text{eff}} \]

Raman response
\[ f_R h_R(T) \]

Can include noise, frequency-dependent mode area, polarization

Validity: down to single cycle regime

Abstract:
We describe generalized nonlinear envelope equation modeling of sub-cycle dynamics on the underlying electric field carrier during one-dimensional propagation in fused silica. Generalized envelope equation simulations are shown to be in excellent quantitative agreement with the numerical integration of Maxwell's equations, even in the presence of shock dynamics and carrier steepening on a sub-50 attosecond timescale. In addition, by separating the effects of self-phase modulation and third harmonic generation, we examine the relative contribution of these effects in supercontinuum generation in fused silica nanowire waveguides.

© 2007 Optical Society of America

OCIS codes:
060.7140 Ultrafast processes in fibers, 190.7110 Ultrafast nonlinear optics

References and links
Modeling pulse propagation

- Generalized nonlinear Schrödinger Equation (GNLSE)

\[
\frac{\partial A}{\partial z} + \frac{\alpha}{2} A - \sum_{k \geq 2} \frac{i^{k+1}}{k!} \beta_k \frac{\partial^k A}{\partial T^k} = i \gamma \left(1 + i \tau_{\text{shock}} \frac{\partial}{\partial T}\right) \left(A(z, t) \int_{-\infty}^{+\infty} R(T') |A(z, T - T')|^2 dT'\right)
\]

- Important considerations
  - window size (avoid wrapping around)
  - temporal/spectral resolution
  - step size (FWM artifacts)
  - DO NOT use linear Raman model
  - Disp. coeff. must be changed if you change the pump wavelength
Modeling supercontinuum generation

- Good agreement with experimental results

- Success in modeling: physics behind supercontinuum generation now well-understood
Supercontinuum generation is easy!

But its dynamics can be rather complex…
Deconstructing supercontinuum dynamics

- 30 fs, 10 kW input pulses

Evolution can be divided in 3 stages
- Initial higher-order soliton compression (spectral broadening),
- soliton fission and dispersive wave generation
- Raman self-frequency shift
Evolution can be divided in 3 stages

- Initial higher-order soliton compression (spectral broadening),
- soliton fission and dispersive wave generation
- Raman self-frequency shift
Deconstructing supercontinuum dynamics

- Understanding the complex dynamics: study separately the different stages

- Soliton propagation dynamics
  - Fundamental solitons
  - Higher-order solitons
  - Effect of perturbation: higher-order dispersion and Raman scattering
Fundamental soliton

- Fundamental soliton is invariant upon propagation (except for a constant nonlinear phase-shift)

\[ A(z = 0, T) = \sqrt{P_0} \text{sech}(T/T_0) \]
\[ A(z, T) = \sqrt{P_0} \text{sech}(T/T_0)e^{jk_{sol}z} \]

Requirements
\[ N = \sqrt{L_d / L_{nl}} = \sqrt{\gamma P_0 T_0^2 / |\beta_2|} = 1 \]

Soliton number: \( \gamma P_0 / 2 \)

- Time evolution
- Spectrum evolution

\( \frac{Z}{Z_{sol}} \)

Time (ps)   Frequency (THz)
Higher-order solitons

- Higher-order soliton is periodic upon propagation: \( A(z + z_{\text{sol}}, T) = A(z, T) \)

Requirements:

\[
A(z = 0, T) = \sqrt{P_0} \text{sech}(T / T_0)
\]

\[
N = \sqrt{L_d / L_{nl}} = \sqrt{\gamma P_0 T_0^2 / |\beta_2|} = 2, 3, 4, \ldots
\]

\[
z_{\text{sol}} = \frac{\pi}{2} L_d = \frac{\pi}{2} T_0^2 / |\beta_2|
\]

Soliton period

Quantized!

- Time evolution
- Frequency (THz)
- Spectrum evolution
- Time (ps)
- dB

\( N = 2 \)
Higher-order solitons

- Higher-order soliton is periodic upon propagation:
  \[ A(z + z_{\text{sol}}, T) = A(z, T) \]

Requirements:

\[
A(z = 0, T) = \sqrt{P_0} \text{sech}(T / T_0)
\]

\[
N = \sqrt{L_d / L_{\text{nl}}} = \sqrt{\gamma P_0 T_0^2 / | \beta_2 |} = 2, 3, 4, ...
\]

\[
z_{\text{sol}} = \frac{\pi}{2} L_d = \frac{\pi}{2} T_0^2 / | \beta_2 |
\]

Soliton period

Quantized!

Time evolution

Spectrum evolution

\[ N = 3 \]

Frequency (THz)

Time (ps)
Cutter in 3D

N=1

N=2

N=3
Perturbations of solitons

- Solitons (fundamental/higher-order)
  - Solutions of the pure nonlinear Schrödinger equation (only GVD and Kerr nonlinearity)
    \[
    \frac{\partial A}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} = i\gamma |A|^2 A
    \]

- Higher-order dispersion and Raman scattering perturb the ideal evolution of solitons
  - Soliton self-frequency shift
  - Dispersive wave generation
  - Soliton fission
    \[
    \frac{\partial A}{\partial z} + i \frac{\beta_2}{2} \frac{\partial^2 A}{\partial T^2} - \frac{\beta_3}{6} \frac{\partial^3 A}{\partial T^3} + \ldots = i\gamma (1 - f_R) |A|^2 A + f_R A (h_R * |A|^2)
    \]
Raman perturbation of N=1 soliton

- Fundamental soliton propagating in the presence of stimulated Raman scattering

- Frequency of soliton shifts towards lower frequencies (longer wavelengths) with propagation = soliton SELF-frequency shift

\[ \nu_0 \]

Frequency shifts scales as \( T_0^{-4} \)
Raman perturbation of N=1 soliton

- Frequency of soliton shifts towards lower frequencies (longer wavelengths) with propagation = soliton SELF-frequency shift

- Parabolic/linear trajectory in the time/frequency domain
HOD perturbation of N=1 soliton

- Dispersion depends on wavelength/frequency
- Soliton near the zero-dispersion wavelength is strongly perturbed: part of its spectrum extends in the normal dispersion regime.
HOD perturbation of N=1 soliton

- Higher-order dispersion: fundamental soliton sheds energy as a dispersive wave
- Dispersive wave located in the normal dispersion regime
  - phase-matching condition: \( \text{phase}^{(\text{soliton})} = \text{phase}^{(\text{disp. wave})} \)

\[
\beta(\omega_{\text{sol}}) - \omega_{\text{sol}} \beta_1(\omega_{\text{sol}}) + \frac{\gamma P_0}{2} = \beta(\omega_{\text{DW}}) - \omega_{\text{DW}} \beta_1(\omega_{\text{sol}})
\]

\[
\frac{\beta_2}{2} \Omega^2 + \frac{\beta_3}{6} \Omega^3 + \ldots = \frac{\gamma P_0}{2}
\]

\[
\Omega \approx \frac{3|\beta_2|}{\beta_3} + \frac{\gamma P_0 \beta_3}{3 \beta_2^2}
\]
HOD perturbation of N=1 soliton

Nonlinear pulse propagation in the neighborhood of the zero-dispersion wavelength of monomode optical fibers

Laboratory for Plasma and Fusion Energy Studies, University of Maryland, College Park, Maryland, 20742

Received February 16, 1986; accepted May 6, 1986

Nonlinear pulse propagation is investigated in the neighborhood of the zero-dispersion wavelength in monomode fibers. When the amplitude is sufficiently large to generate breathers (N > 1 solitons), it is found that the pulses break apart if λ = λ0 is sufficiently small, owing to the third-order dispersion. Here λ0 denotes the zero-dispersion wavelength. By contrast, the solitary-wave (N = 1) solution appears well behaved for arbitrary λ = λ0. Implications for communication systems and pulse compression are discussed.

Cherenkov radiation emitted by solitons in optical fibers

Nail Akhmediev and Magnus Karlsson*
Optical Sciences Centre, Australian National University, Canberra, Australian Capital Territory 0200, Australia
(Received 15 April 1994)

We demonstrate a simple, fully analytic method of calculating the amount of radiation emitted by optical solitons perturbed by higher-order dispersion effects in fibers and find good agreement with numerical results. It is pointed out that this radiation mechanism is analogous to the well-known Cherenkov radiation processes in nonlinear optics.
Higher-order dispersion: fundamental soliton sheds energy as a dispersive wave

Dispersive wave located in the normal dispersion regime
  - phase-matching condition: \( \text{phase}^{(\text{soliton})} = \text{phase}^{(\text{disp. Wave})} \)
Long means short…

- Longer soliton wavelength = shorter dispersive wave wavelength

- Energy of dispersive wave inversely proportional to soliton detuning with respect to ZDW
Soliton fission

- Higher-order $N$-soliton is unstable, sensitive to perturbations
  - $N$-soliton breaks up into $N$ fundamental solitons

\[ A(z = 0, T) = \sqrt{P_0} \text{sech}(T/T_0) \]

\[ \frac{L_D}{L_{NL}} = \frac{\gamma P_0 T_0^2}{|\beta_2|} = N^2 \]

\[ N = 2, 3, 4, 5, ... \]

\[ A_j(T) = \sqrt{P_j} \text{sech}(T/T_j) \]

\[ T_j = \frac{T_0}{2N - 2j + 1} \]

\[ P_j = \frac{(2N - 2j + 1)^2}{N^2} P_0 \]

\[ \frac{\gamma P_j T_j^2}{|\beta_2|} = 1 \]

\[ P_0 T_0 = \sum_{j=1}^{N} P_j T_j \]
Soliton fission

- Higher-order $N$-soliton is unstable, sensitive to perturbations.
  $N$-soliton breaks up into $N$ fundamental solitons.
Raman perturbation of $N$-Soliton
Fission of $N$-soliton

- Fission length: $L_{\text{fiss}} \approx L_d / N = N L_{\text{nl}}$
Let’s put everything together

- Soliton fission + dispersive wave radiation + soliton self-frequency shift = supercontinuum generation
Increasing the bandwidth

- Red shifting soliton interact with the dispersive wave through XPM
- Dispersive wave “trapping” which is blue-shifted
Dispersive wave trapping

- Continuously redshifting soliton induces a continuous blueshift of the dispersive wave
Normal dispersion regime

- Solitons only exist in anomalous dispersion regime: different dynamics
- Self-phase modulation, four-wave mixing
Pump duration

- Initial stages of supercontinuum generation strongly depend on pump pulses duration
- Short pulses (typically $T_0 \leq 150$ fs): soliton dynamics
- Long pulses: modulation instability (four-wave mixing) followed by soliton dynamics
Supercontinuum in the incoherent regime

- Initial dynamics: noise-seeded modulation instability
- Random noise → “chaotic” field
Fiber with two zero-dispersion wavelengths

Time-frequency analysis

- Ultrafast dynamics can be conveniently visualized in the time-frequency domain

Time-frequency analysis
Experimental implementation

\[ E(t, \tau) \propto E(t) E_R(t + \tau) \]

\[ I(\omega, \tau) \propto \int_{-\infty}^{+\infty} E(t) E_R(t + \tau) e^{j\omega t} dt \]

X Frequency Resolved Optical Gating

Short ref. pulse \( E(\omega, \tau) \)
One more thing

Reference pulse duration determines the resolution!

\[ \Delta t \Delta f \geq 1/2 \]
Spectrogram of supercontinuum

- Time-spectrum representation allows to conveniently identify dynamics
Coherence of supercontinuum

- Spectral coherence: “How spectra differs from shot to shot?”

\[
g_{12}^{(1)}(\lambda) = \frac{\langle E_i(\lambda)E_j^{*}(\lambda) \rangle_{i\neq j}}{S(\lambda)}
\]

Overall degree of coherence

\[
|g_{12}^{(1)}| = \frac{\int_{0}^{+\infty} g_{12}^{(1)}(\lambda)S(\lambda)d\lambda}{\int_{0}^{+\infty} S(\lambda)d\lambda}
\]
Coherence of supercontinuum

● Spectral coherence: “How spectra differs from shot to shot?”

\[ g_{12}^{(1)}(\lambda) = \frac{\left\langle E_i(\lambda)E_j^*(\lambda) \right\rangle_{i=\infty}}{S(\lambda)} \]

Overall degree of coherence

\[ |g_{12}^{(1)}| = \frac{\int_{0}^{+\infty} g_{12}^{(1)}(\lambda)S(\lambda)d\lambda}{\int_{0}^{+\infty} S(\lambda)d\lambda} \]

1: coherent
0: incoherent
Coherence of supercontinuum

BUT supercontinuum is not necessarily coherent!

Spectral coherence function

\[ g_{12}^{(1)}(\lambda) = \frac{\langle E_i(\lambda)E_j^*(\lambda) \rangle_{i\neq j}}{S(\lambda)} \]

Overall degree of coherence

\[ |g_{12}^{(1)}| = \frac{\int_0^{+\infty} g_{12}^{(1)}(\lambda)S(\lambda)d\lambda}{\int_0^{+\infty} S(\lambda)d\lambda} \]

1: coherent
0: incoherent
### Regimes of coherence

<table>
<thead>
<tr>
<th></th>
<th>Short pulse regime</th>
<th>Long pulse regime</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Anomalous</strong></td>
<td>- Soliton dynamics</td>
<td>- Modulation instability</td>
</tr>
<tr>
<td>dispersion</td>
<td>- Dispersive waves</td>
<td>- Soliton dynamics</td>
</tr>
<tr>
<td><strong>Normal</strong></td>
<td>- Self-phase modulation</td>
<td>- Four-wave mixing</td>
</tr>
<tr>
<td>dispersion</td>
<td>- Four-wave mixing</td>
<td>- Raman scattering</td>
</tr>
<tr>
<td></td>
<td>- Soliton dynamics</td>
<td>- Soliton dynamics</td>
</tr>
</tbody>
</table>
Measuring supercontinuum coherence

- Michelson interferometer
- Spectral interference
- Fringes visibility gives the coherence function
Fibers with all normal dispersion

- Fibers with normal dispersion at all wavelengths: ANDI
- Allows for high coherence and stability, flat spectra
- Can reach octave-spanning

Coherent octave spanning near-infrared and visible supercontinuum generation in all-normal dispersion photonic crystal fibers

Alexander M. Heidt, Alexander Hartung, Gurthwin W. Bosman, Patrizia Krok, Erich G. Rohwer, Heinrich Schwoerer, and Hartmut Bartelt

14 February 2011 / Vol. 19, No. 4 / OPTICS EXPRESS 3775
Novel glasses for mid-IR

- Use of materials with low losses in the mid-IR

Fluoride (ZBLAN) fibers (not PCFs)

Material ZDW 1.6 microns

\[ n^2 = n^2_{\text{silica}} \]

Material data:
- SC Average Power ~ 1.3 W
- SC Average Power ~ 23 mW

Novel glasses for mid-IR

- Enhanced nonlinearity
- Engineer dispersion

**Tellurite fibers (PCFs)**

Material ZDW 2.2 microns

\[ n_2 = n_2^{\text{silica}} \times 10^{-20} \]

Novel glasses for mid-IR

- Enhanced nonlinearity
- Engineer dispersion

**Tellurite fibers (PCFs)**

Material ZDW 2.2 microns

\[ n_2 = n_2^{\text{silica}} \times 10^{-20} \]

---


Novel glasses for mid-IR

- Enhanced nonlinearity
- Engineer dispersion

Sulfide/Chalcogenide fibers (PCFs or not)

Material ZDW 5 microns

\[ n_2 = n_{2\text{silica}} \times 100 \]

Novel glasses for mid-IR

- Enhanced nonlinearity
- Engineer dispersion

Sulfide/Chalcogenide fibers (PCFs or not)

Material ZDW 5 microns

\[ n_2 = n_{2\text{silica}} \times 100 \]

State-of-the-art

- Many pumping schemes and SC characteristics possible
Initial stages of supercontinuum generation strongly depend on pump pulses duration.

- Short pulses (typically $T_0 \leq 150$ fs): soliton dynamics
- Long pulses: modulation instability (four-wave mixing) followed by soliton dynamics
Supercontinuum instabilities

- Long pulse regime

- Early stage: noise seeded modulation instability
  - Growth of symmetrical sidebands around the pump
  - Pulse breaks into a random train of shorter pulses
Supercontinuum instabilities

- Noise-seeded MI: large pulse to pulse instabilities
- Commonly seen for > 150 fs pulses
- Leads to artificial spectral smoothness
Fluctuations in supercontinuum generation

Already known in early 2000…

- 2007: real-time detection of these fluctuations on the long wavelengths
- Dispersion maps wavelength to time
- Filtering selects long wavelength edge

Gu et al., OL (2002)
Dudley et al., JSTQE (2002)
Corwin et al., PRL (2003)
Dudley et al., RMP (2006)
Fluctuations in supercontinuum generation
Fluctuations in supercontinuum generation

- “Optical rogue waves” by analogy with hydrodynamics

Presence of extreme events

Skewed statistical distribution
Mind the filter…

High-intensity bin event
Mind the filter…

Filter distorts statistics: rogue events are only “rogue” in amplitude because of the filter.

Low-intensity bin event
Insight from numerics

- Numerically possible to detect ALL solitons from ALL shots

- Only few events satisfy hydrodynamic rogue wave criterion

Erkintalo et al., EPJ Special Topics 185, 135 (2010)
Collisions are more extreme

Soliton-collision are the most powerful waves and satisfies hydrodynamic "rogue" wave criterion

Erkintalo et al., EPJ Special Topics 185, 135 (2010)
Mussot et al., Opt. Exp. 17, 17010 (2009)
The subtle role of collisions

- In fact, the filtering technique indirectly capture collision events (to some extent)

- Collisions of solitons lead to energy exchange and enhanced redshift (the larger sucks energy from the smaller)
BTW phase matters

- Collision is phase-dependent

Antikainen et al., Nonlinearity 25, R73 (2012)

- Soliton-collision is not necessarily extreme
Capturing single shot spectra

- Real-time measurement of single shot is possible
- Use dispersive time-to-frequency transformation

- At large distance in the dispersive fiber, the stretched temporal trace corresponds to the spectrum: analog to far-field diffraction

Goda et al., Nature Photonics 7, 102–112 (2013)
Capturing single shot spectra

- Real-time measurement of single shot confirms large fluctuations
Capturing single shot spectra

- Direct access to spectral fluctuations at ALL wavelengths

Wetzel et al., Scientific Reports 2, 882 (2012)
Link with traditional discrete nonlinear optics

- Supercontinuum: described in terms of pulse dynamics in the time-domain

- Spectrum of mode-locked lasers consists of discrete modes
Link with traditional discrete nonlinear optics

- We should be able to interpret SC generation in terms of traditional discrete wave mixing dynamics

- Dispersive wave generation: only phase matching condition, energy conservation?
Cascaded four-wave mixing

- Two CW pumps lead to the generation of multiple discrete sidebands through cascaded four-wave mixing.
Cascaded four-wave mixing

- Two CW pumps lead to the generation of multiple discrete sidebands through cascaded four-wave mixing.
Cascaded four-wave mixing

- Two CW pumps lead to the generation of multiple discrete sidebands through cascaded four-wave mixing

\[ \chi^{(2n+1)} \]

- The net result is a \( \chi^{(2n+1)} \) Process!

\[ \omega_{+p} + \omega_{n-1} \rightarrow \omega_{-p} + \omega_{n} \]

\[ (n+1)\omega_{+p} \rightarrow n\omega_{-p} + \omega_{n} \]

BUT: only efficient energy transfer if the process is phase-matched
Cascaded four-wave mixing

• Such cascaded four-wave mixing is naturally phase-matched in optical fibers due to the symmetry breaking!

\[
(n + 1)\omega_{+p} \rightarrow n\omega_{-p} + \omega_n \quad \text{Taylor-series expansion}
\]

\[
(n + 1) \beta(\omega_{+p}) = n \beta(\omega_{-p}) + \beta(\omega_n) \quad \Rightarrow \quad \omega_n = \omega_0 + 3|\beta_2|/\beta_3
\]

Only phase-matched through the cascade!

• Leads to asymmetry in the cascaded spectrum

2 pumps: anomalous dispersion
Phase-matched side-band: normal dispersion
Precisely the phase-matching condition for dispersive wave generation by solitons!

\[ \omega_n = \omega_0 + 3|\beta_2|/\beta_3 \quad \text{and} \quad \omega_{\text{DW}} = \omega_S + 3|\beta_2|/\beta_3 \]

Cascaded four-wave mixing: discrete frequency-domain description of dispersive wave generation by solitons
Experimental evidence

Erkintalo et al., PRL 109, 223904 (2012)
Future challenges

- New spectral regions
  - UV < 350 nm: absorption & dispersion
  - mid-IR >4/5 microns: absorption

Sylvestre et al., OL 37, 130-132 (2012)

Stark et al., OL 37, 770-772 (2012)

Joly et al., PRL 106, 203901 (2011)
Future challenges

- Stable SC from long pump pulses
  - Seeding modulation instability

Cheung et al., OL 36, 160-162 (2011)

Solli et al., PRL 101, 233902 (2008)
It’s all in there…
Supercontinuum in water waves

- Similar NLS equation governs deep water waves dynamics
  \[
  i \left( \frac{\partial A}{\partial x} + \frac{2k_0}{\omega_0} \frac{\partial A}{\partial t} \right) - \frac{k_0}{\omega_0^2} \frac{\partial^2 A}{\partial x^2} - k_0^3 |A|^2 A = 0.
  \]

- Perturbations also lead to soliton fission!

Hydrodynamic supercontinuum